Section 4.3: Logarithmic Functions

Video 1

The ***logarithm*** $\log_a x$ is the power that *a* must be raised to in order to equal *x*.

The number *a*, $a > 0$ and $a \neq 1$, is called the ***base*** of the logarithm.

The positive number *x* is called the ***argument*** of the logarithm.

An equation is in ***logarithmic form*** if it is written in this form: $y = \log_a x$.

An equation is in ***exponential form*** if it is written in this form: $a^y = x$.

The two forms, $y = \log_a x$ and $a^y = x$, are equivalent equations.

1) Convert from one form to the other.

Logarithmic Form	**Exponential Form**
a)	$3^2 = 9$
b) $\log_2 16 = 4$	
c)	$10^5 = 100,000$
d) $\log_5 \left(\frac{1}{125}\right) = -3$	
e)	$7^1 = 7$
f) $\log_9 1 = 0$	

Video 2

2) Solve the logarithmic equation. (Begin by rewriting the equation in exponential form.

a) $\log_x\left(\frac{25}{49}\right) = -2$

b) $\log_{32} x = \frac{4}{5}$

c) $\log_{81}\sqrt{3} = x$

Video 3

A ***logarithmic function*** is a function of the form $f(x)=\log_a x$, where $a>0$ and $a\neq 1$.

3) Find the inverse function of $f(x)=3^x$.

4) Find the inverse function of $f(x)=\log_a x$.

So, exponential and logarithmic functions are inverses of each other.

Video 4

For $a > 1$:

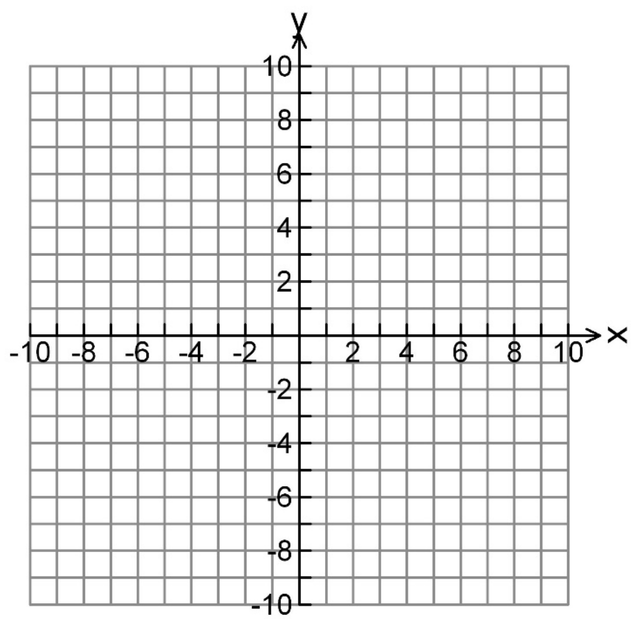
The graph of a logarithmic function $f(x) = \log_a x$ is increasing over its entire domain $(0, \infty)$.

The range of the function is $(-\infty, \infty)$.

It has a vertical asymptote on the *y*-axis $(x = 0)$.

It passes through the points $\left(\frac{1}{a}, -1\right)$, $(1, 0)$, and $(a, 1)$.

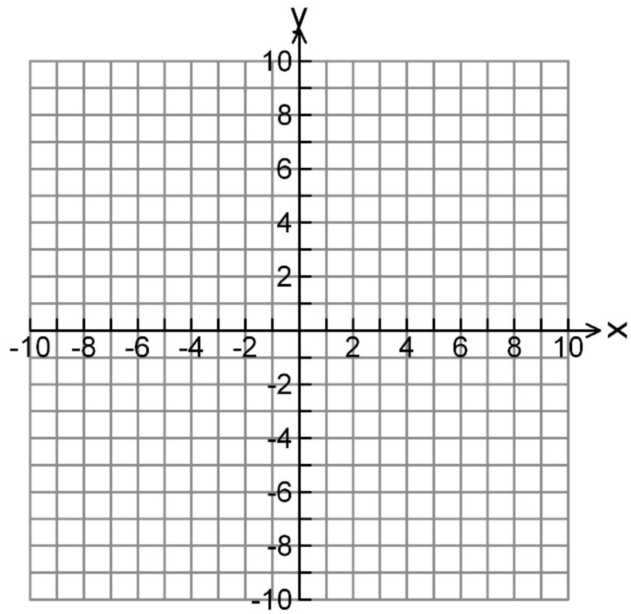
4) Graph $f(x) = \log_3 x$.



Compare this graph and the process used to make it to the graph of the exponential function $f(x) = 3^x$

For $0 < a < 1$, the graph of $f(x) = \log_a x$ is similar to that where $a > 1$, except the function is decreasing over its entire domain.

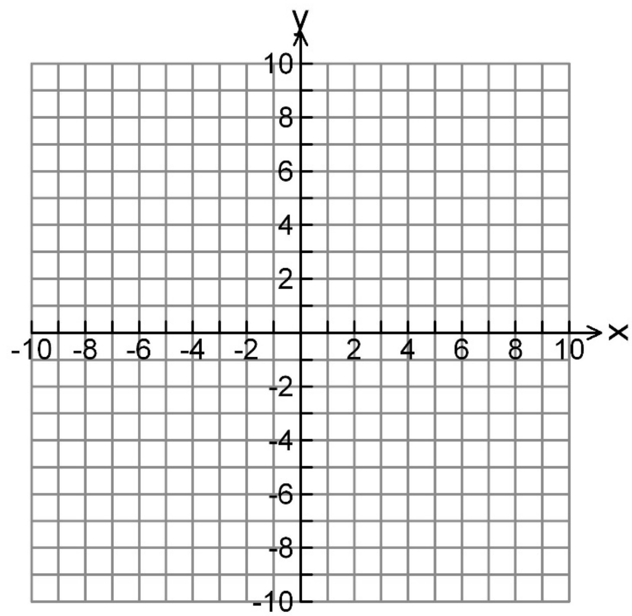
5) Graph $f(x) = \log_{1/4} x$.



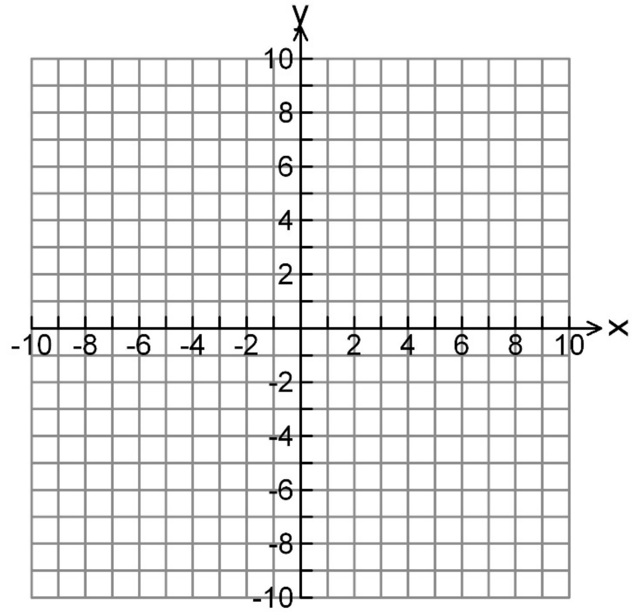
Video 4

To graph a function of the form $f(x) = \log_a(x-h)+k$, begin with the graph of the basic function $f(x) = \log_a x$, and apply a horizontal shift of *h* units and a vertical shift of *k* units.

6) Graph $f(x) = \log_2(x-5)+3$.



7) Graph $f\left(x\right)=\log_{1/3}\left(x+1\right)-2$.



Video 5

Properties of Logarithms:

$\log_a 1 = 0$

$\log_a a = 1$

Product Property: $\log_a (x \cdot y) = \log_a x + \log_a y$

Quotient Property: $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$

Power Property: $\log_a x^r = r \cdot \log_a x$

$a^{\log_a x} = x$

$\log_a a^x = x$

8) Expand.

a) $\log_a\left(x^3y^2\sqrt{z}\right)$

b) $\log_a\left(\frac{xy^3}{z^4w^5}\right)$

c) $\log_a\sqrt[m]{\frac{r^3}{s^4t^7}}$

9) Condense.

a) $\log_a x - \log_a y + \log_a z$

b) $3\log_a x + 4\log_a y - 5\log_a z - \frac{1}{2}\log_a z$

c) $2 - \log_a x + 9\log_a y$

10) Given that $\log_{10} 7 \approx 0.845$, find the following.

a) $\log_{10} 343$

b) $\log_{10}\left(\frac{100}{7}\right)$